

Comment on "Classification scheme for phenomenological universalities in growth problems in physics and other sciences"

Dibyendu Biswas^{1*} and Swarup Poria^{2†}

¹Department of Basic Science, Humanities and Social Science (Physics)

Calcutta Institute of Engineering and Management

24/1A Chandi Ghosh Road, Kolkata-700040, India

²Department of Applied Mathematics, University of Calcutta

92 Acharya Prafulla Chandra Road, Kolkata-700009, India

Castorina *et al.* [1, 2] have attempted to derive logistic equations based on the formalism of phenomenological universalities, found in different field of sciences. They suggested that the logistic equations can be derived, for negative b , from the equation given by,

$$\frac{dy}{d\tau} = \alpha_2 y^p - \beta_2 y \quad (1)$$

where, $\alpha_2 = (1 + b)/b$, $p = 1 - b$ and $\beta_2 = 1/b$. The usual logistic growth equation is obtained for $p = 2$ [1, 2]. Here we point out that the conditions do not fulfill the characteristic features of logistic growth equation. The changes take place in the coefficients (α_2 and β_2) with the change in p (or b) is not considered here. The usual logistic growth equation is expressed generally in terms of *carrying capacity* that is not considered in this phenomenological description. Even logistic growth equations do not belongs to the phenomenological class described as U2 [1, 2]. Different cases obtained for different values of p ((for $b < 0$) are considered in the following section.

The condition $p = 2$ follows $b = -1$, $\alpha_2 = 0$ and $\beta_2 = -1$. The characteristic equation, derived from equation (1), of corresponding system is as follows,

$$\frac{dy}{d\tau} = y \quad (2)$$

The condition $b < -1$ implies that $\alpha_2 > 0$, $p > 2$ and $-1 < \beta_2 < 0$. The governing equation of the corresponding system is

*dbbesu@gmail.com

†swarupporia@gmail.com

$$\frac{dy}{d\tau} = \alpha_2 y^p + |\beta_2| y \quad (3)$$

The conditions $\alpha_2 < 0$, $1 < p < 2$ and $\beta_2 < -1$ are valid for $-1 < b < 0$. The conditions immediately follow,

$$\frac{dy}{d\tau} = |\beta_2| y - \alpha_2 y^p \quad (4)$$

Equation (2) and (3) do not represent logistic equations. Apparently it appears like that the equation (3) follows logistic growth equations, but the associated condition $a(0) = 1$ is in contradiction with the characteristic feature of logistic equation, that is also true for equation (2) and (3).

We have repeated the calculation of the phenomenological class, as proposed by Castorina *et al.*, $\varphi = b_1 a + b_2 a^2$ (instead of $\varphi = a + b a^2$), representing the class in a more generalized manner, with the assumption that $y(0) = 1$ and $a(0) = b_1(1 - \frac{1}{K})$; where K is the carrying capacity. It is found that the system follows,

$$\frac{dy}{d\tau} = \alpha y^\sigma - \beta y \quad (5)$$

Where, $\sigma = 1 - b_2$, $\alpha = b_1(1 + \gamma)/b_2$, $\gamma = b_2(K - 1)/K$ and $\beta = b_1/b_2$; with the solution,

$$y = [1 + \gamma - \gamma \exp(-b_1 \tau)]^{1/b_2} \quad (6)$$

It is easy to show that equation (5) and (6) correspond to the well-known West-

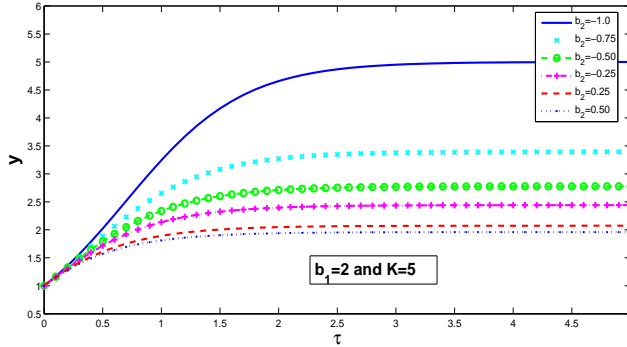


Figure 1: Growth curves for $K = 5$ and $b_1 = 2$. From the top to the bottom the values of the parameter b_2 are -1.0 , -0.75 , -0.5 , -0.25 , 0.25 , 0.5 . The solid curve ($b_2 = -1.0$) corresponds to the usual logistic growth, while the dashed line ($b_2 = 0.25$) represents biological growth following West-type equation.

type equation of biological growth [3] for the condition $b_1 > 0$ and $b_2 = 0.25$. Other biological growth processes [4] may be described for $0 < b_2 < 0.33$. The

same leads to logistic equation for the condition $b_1 > 0$ and $b_2 < 0$. The usual logistic equation is found for $b_2 = -1$. When $\gamma = -1$, the system follows exponential growth for which $b_2 < 0$ for $K > 1$. Such behavior is not observed for $b_2 > 0$.

Therefore, the phenomenological class described above is not similar to the U2 class [1, 2]. It is because of the fact that φ is similar in nature with U2, but the initial conditions are different from U2. In brief, we have defined a new class of phenomenological universalities that leads to logistic equation as well as West-type biological growth equation based on the values of coefficients of phenomenological description.

References

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